$$\lim_{x\to 3} \frac{2}{x+3} = \frac{1}{3}$$

To prove

Given: $\varepsilon > 0$

Define: $\delta = \min\{1, 15\varepsilon\}$

Assume

$|x-3| < \delta$

$$\frac{|x-3|}{3|x+3|} < \frac{|x-3|}{15} < \frac{\delta}{15} = \frac{15\varepsilon}{15} = \varepsilon$$

$$\frac{|3-x|}{|3(x+3)|} < \varepsilon$$

$$\frac{|6-(x+3)|}{|3(x+3)|} < \varepsilon$$

$$\left|\frac{2}{x+3} - \frac{1}{3}\right| < \varepsilon$$

$$\therefore \lim_{x\to 3} \frac{2}{x+3} = \frac{1}{3}$$

$|x-3| < \delta$

$$\left|\frac{2}{x+3} - \frac{1}{3}\right| < \varepsilon$$

$$\left|\frac{2\cdot 3}{(x+3)3} - \frac{1(x+3)}{3(x+3)}\right| < \varepsilon$$

$$\left|\frac{6-x-3}{3(x+3)}\right| < \varepsilon$$

$$\frac{|3-x|}{|3(x+3)|} < \varepsilon$$

$$\frac{|x-3|}{3|x+3|} < \varepsilon$$

$$\frac{1}{3|x+3|}|x-3| < \varepsilon$$

$\delta = 15\varepsilon$

$$\frac{x1}{21} < \frac{1}{3|x+3|} < \frac{1}{15}$$

$$\lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$$

<u>To Prove:</u>

Given $\varepsilon > 0$

Define $\delta = \min\{1, 2\varepsilon\}$

Assume $|x-2| < \delta$

$$\frac{|x-2|}{|2x|} < \left|\frac{x-2}{2}\right| < \frac{\delta}{2} = \frac{\cancel{2}\varepsilon}{\cancel{2}} = \varepsilon$$

$$\frac{|2-x|}{|2x|} < \varepsilon$$

$$\left|\frac{1}{x} - \frac{1}{2}\right| < \varepsilon$$

$$\therefore \lim_{x \to 2} \frac{1}{x} = \frac{1}{2}$$

S.W.

$$|x-2| < \delta$$

$$\left|\frac{1}{x} - \frac{1}{2}\right| < \varepsilon$$

$$\left|\frac{2-x}{2x}\right| < \varepsilon$$

$$\frac{|x-2|}{|2x|} < \varepsilon$$

$$\left|\frac{1}{2x}\right| |x-2| < \varepsilon$$

$$\frac{1}{6} < \frac{1}{|2x|} < \frac{1}{2}$$

$$\frac{\delta}{2} = \varepsilon$$

$$\delta = 2\varepsilon$$

$$\lim_{x \to 4} \sqrt{x} = 2$$

$$|x-4| < \delta$$

$$|\sqrt{x} - 2| < \varepsilon$$

$$\left| \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} \frac{\sqrt{x}-2}{1} \right| < \varepsilon$$

$$\frac{|x-4|}{|\sqrt{x}+2|} < \varepsilon$$

$$\delta = (\sqrt{3}+2)\varepsilon$$

$$* \ \frac{1}{\sqrt{5}+2} < \frac{1}{\sqrt{x}+2} < \frac{1}{\sqrt{3}+2}$$